ME 7120 FEA

Project 3

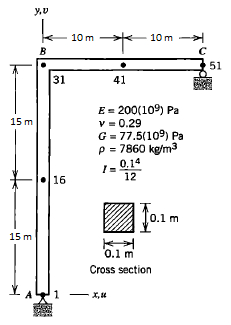
Newmark Method

by

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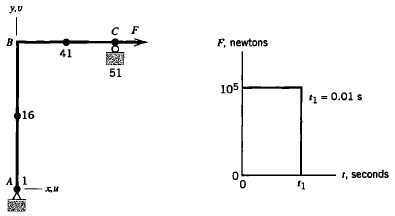
Introduction

This report discusses the results obtained from the use of Newmark method in order to solve a L-beam problem that is loaded by an impulse with a magnitude of 100,000 N. Figure 1 shows the definition of the structure.



**Figure 1**: Structure and material properties.

The structure is comprised of 51 nodes and 50 2-D beam elements, each of length 1m. The structure is pinned at location A and simply supported at location C such that displacement in y-dir. is not allowed. The structure is loaded by an impulse load shown in Figure 2.

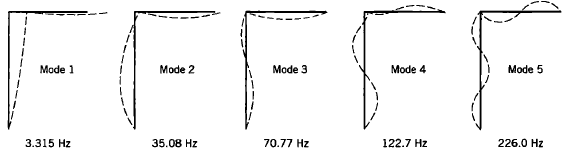


**Figure 2**: Loading profile for the structure.

Stiffness (**K**) and mass (**M**) matrices are generated using 2 nodded beam element from project #1. Since the problem defined in Figure 1 is in 2-D space the unnecessary degrees of freedom, which is in this case are displacement in y-dir and rotations about x and z, were removed. Therefore, the size of **K** and **M** matrices is reduced from 306X306 to 150X150. The degrees of freedom that were kept are displacements in x and z directions along with rotation about y-axis for each node of the system.

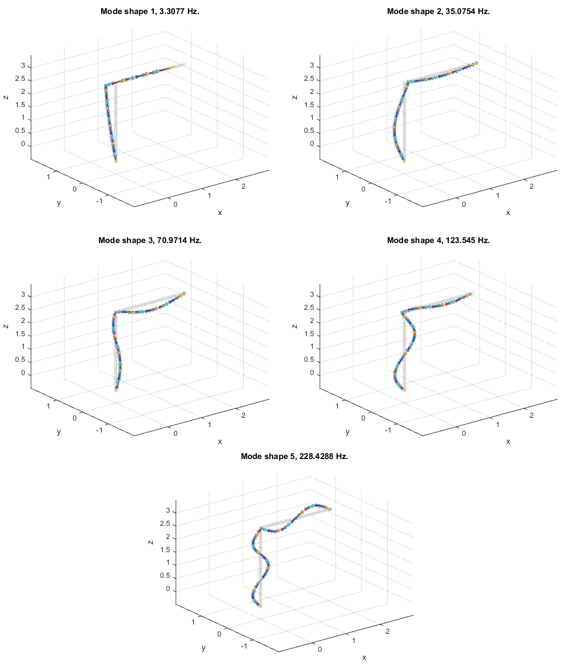
Model Validation

The reduced matrices were used to calculate the first 5 natural frequencies along with their respective mode shapes. This was done in order to compare the results found in the FEA book by Cook. Results from the book can be seen in Figure 3.



**Figure 3**: First 5 modes from the L-bracket (Book Results)

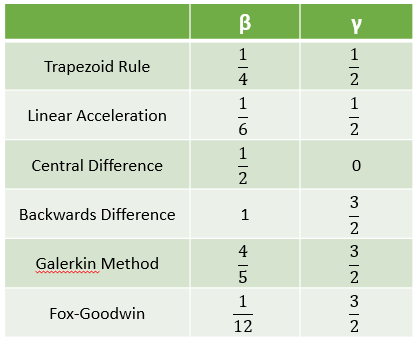
Results from the reduced matrices can be seen in Figure 4. It can be seen that the results obtained from the reduced **M** and **K** closely match the book results. Based on this evaluation it was deemed sufficient to use the derived model for the dynamic loading analysis based on the loading profile seen in Figure 2.



**Figure 4**: First 5 modes for the L-bracket (reduced **M** and **K**)

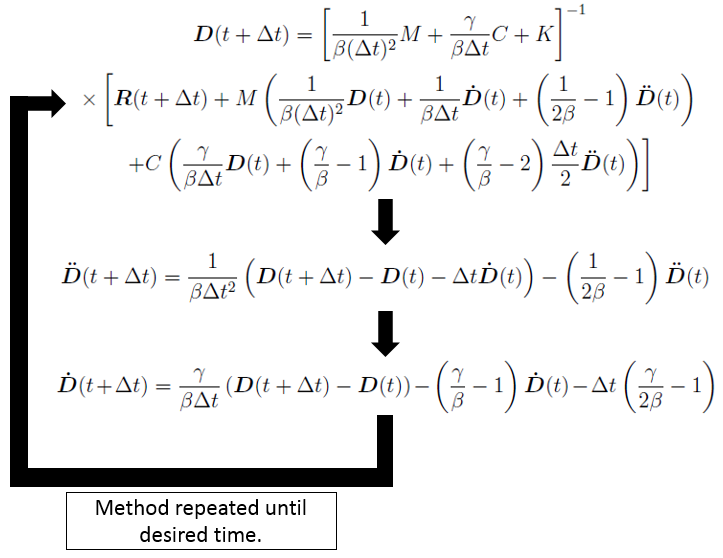
Dynamic Loading Analysis

For the dynamic analysis Newmark method was used. Table 1 shows the 5 different combinations of β and γ that yield different methods.



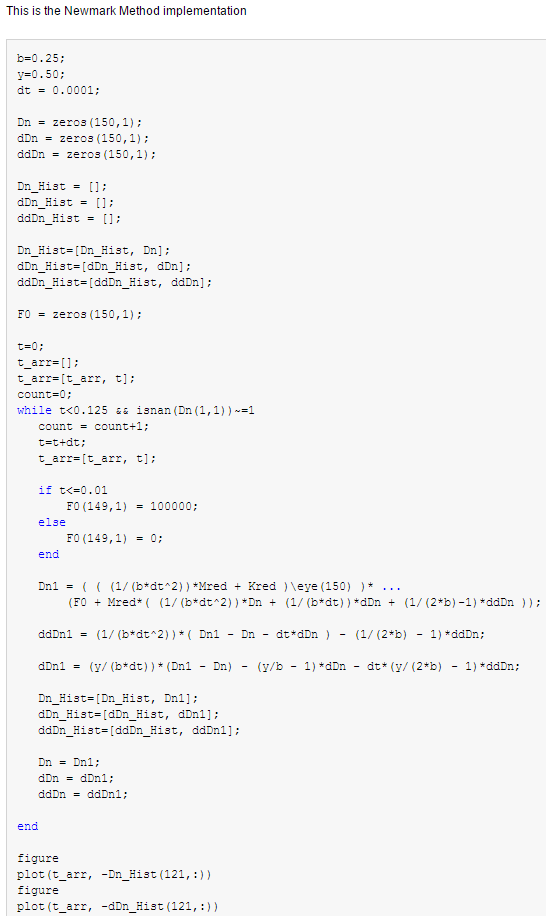
**Table 1**: List of methods used to calculate structure response.

This combination of β and γ was implemented in the Newmark method that is outlines in Figure 5 below. The implementation did not use any damping.



**Figure 5**: Newmark method outline

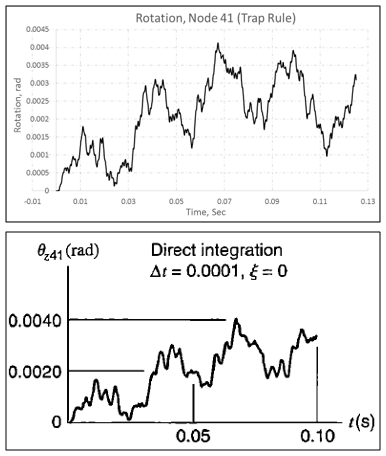
Method outlined in figure 5 was coded in Matlab where the code is shown below in figure 6. For initial starting conditions it is assumed that displacement, velocity, and acceleration are all 0. The load is applied to the 51’s node and will be applied up until time (t) reaches 0.01 seconds. The total run time for the simulation is 0.125 seconds.



**Figure 6**: Newmark method implemented in Matlab.

Results

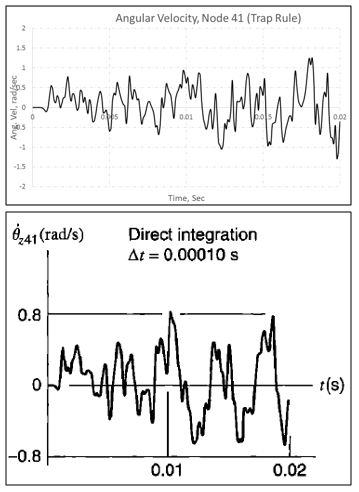
First, the results for the Trapezoidal method is presented where β = 0.25 and γ = 0.5. These results are compared to those found in the book. In order to match the results in the book a ∆t = 0.0001 sec, which was given in the book. Figure 7 compares the results for the rotation of node 41.



**Figure 7**: Rotation vs time at Node 41. a) Solution plot. b) book results

Solution result shows more detail in the time response but it matches quite well with the results from the book. In the time range from 0.0 to 0.1 sec there are 4 peaks, which are similar between the two plot. Additionally, the magnitude of the node rotation is the same between the two plots.

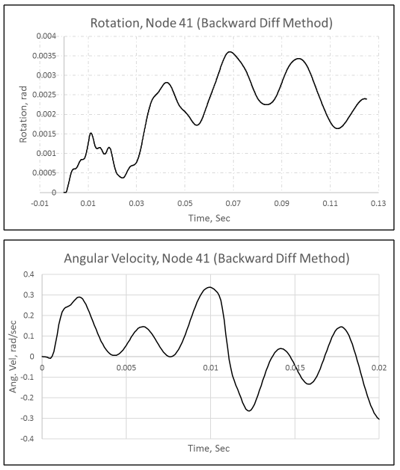
In figure 8 the plots show the rotational velocity of node 41. In similar fashion the solution results show mode detail compared to the book plot. However, the primary features are matching quite well. In addition, the magnitude of the rotational velocity is similar between the two plots.



**Figure 8**: Rotational velocity vs time at Node 41. a) Solution plot. b) book results

Second, the results for the backward difference method is presented where β = 1.0 and γ = 1.5. Time step is maintained at ∆t = 0.0001 sec. There are no results in the book to compare to but it is interesting to note the differences compared to the Trap method.

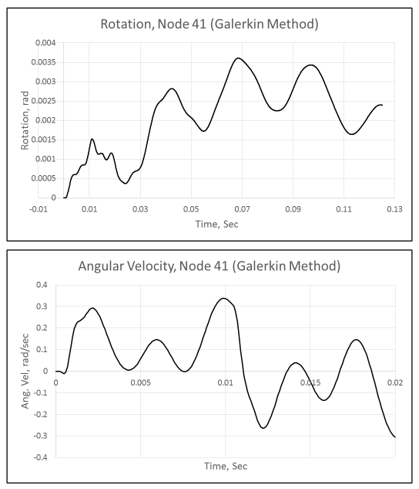
Comparing figures 7, 8, and 9 it is evident that the response plotted using the backwards difference method is much smoother when compared to the trap method. It appears that there is some artificial damping being provided when using the backward difference method. Despite this fact the amplitudes in the plots are relatively close.



**Figure 9**: Rotation and rotational velocity at node 41 using backward difference method.

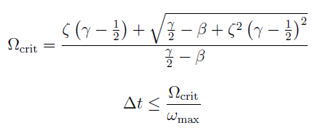
Third, the results for the Galerkin method is presented where β = 0.8 and γ = 1.5. Time step is maintained at ∆t = 0.0001 sec. There are no results in the book to compare to but it is compared to the Trap method and the backward difference method.

The Galerkin and backward difference methods give pretty much the same answer. They both appear to be smoother when compared to the trap method. As stated previously, this is possibly attributed to the artificial damping provided by using the selected β and γ values.



**Figure 10**: Rotation and rotational velocity at node 41 using Galerkin method.

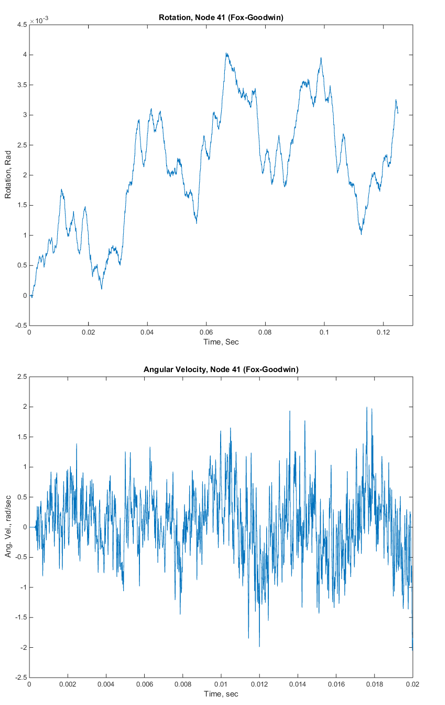
In addition, Linear acceleration and central difference were attempted. However, the solutions were unstable and the system solution diverged with increasing time. For both of the above mentioned cases a ∆t = 0.0001 sec was used, which at first glance would indicate that the time step is not sufficiently small. In order to calculate the sufficient ∆t, we use the equations listed below.



**Figure 11**: Stability conditions

Using the expressions in Figure 11, the required ∆t for linear acceleration is 0.000029 sec. This time step was implemented but it still yielded a diverging solution. Exploring smaller time steps is not feasible since the solution time unreasonably long. For the central difference method no is defined. Therefore it is not possible to estimate a stable ∆t.

Finally, at the last minute, decision was made to try the Fox-Goodwin method. For this case a ∆t of 0.000001 sec was used in order to insure a converged solution. The resulting rotation and rotational velocity plots match closely to the results obtained using the Trap method. These plots can be seen in Figure 12.



**Figure 12**: Solution results using Fox-Goodman.